

# CROSS CORRELATION IN PHASE NOISE ANALYSIS

Phase noise is a property of an oscillator that can extend in magnitude from the carrier of several volts down to a mere nano-volt far from the carrier. In many cases the lowest noise OCXOs, SAWs and other specialty oscillators have carrier to noise ratios in excess of -180 dBc/√Hz. The noise level of these oscillators often extends below that of even the mixers and low noise amplifiers at baseband. Cross correlation is a method used in phase noise analysis to extend the range of any single channel measurement by introducing a second channel and utilizing signal processing to locate the noise that is common to the DUT, yet uncommon to each individual channel. With this method, a typical noise floor improvement of 20 dB is very realistic, allowing for high accuracy measurements of extremely low noise oscillators. This article presents the mathematics with an example of how cross correlation can accurately identify signals or noise that is below the level of the measurement instrument.

Most phase noise measurement systems use what is called carrier cancellation. Phase noise is not measured directly, but down-converted to baseband. In an absolute phase noise measurement, where the absolute noise level of an oscillator is being measured, two oscillators are phase locked to one another. Once the two signals are locked with a mixer, the phase noise

of both channels is down-converted directly to baseband without the carrier which is at DC and cancelled.

In a residual or additive phase noise measurement, whereby the additive noise of a component such as an amplifier is to be measured, the oscillator is split into two parts. One path drives the LO port of the mixer while the other path goes through the DUT prior to going into the RF port of the mixer. Within the noise level and isolation of the mixer, the carrier and its noise are canceled, being common between the two paths, while the noise of the DUT is measured directly at an offset and its carrier frequency centered at DC.

In both cases, low noise, low frequency techniques are then applied to amplify and sample this signal. However, in both cases, noise levels of system components may limit the measurement dynamic range or noise floor. In absolute measurements, the reference oscillator typically is the limitation. In additive measurements, even a very good mixer can often contribute as much noise as a low noise DUT.

Cross correlation has been used by NIST for metrology level phase noise measurements for quite some time (see the references for more information or go to [www.nist.gov](http://www.nist.gov)). Through-

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JASON BREITBARTH

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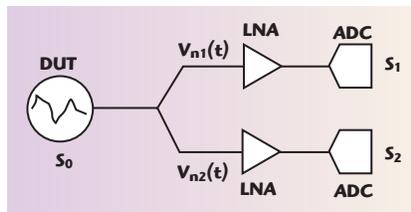
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**TABLE I**  
VOLTAGE NOISE IN TERMS OF  $V_{rms}/\sqrt{Hz}$  AND  $dBV$  FOR VARIOUS RESISTANCES

Resistance ( $\Omega$ )	Voltage Noise @300k ( $91 nV_{rms}/\sqrt{Hz}$ )	$dBV$ ( $dBV_{rms}/\sqrt{Hz}$ )
50	0.91	-180.8
100	1.29	-177.9
10K	4.07	-167.8



▲ Fig. 1 Block diagram of noise measurement system where one signal with noise is measured simultaneously by two channels with their own independent and uncorrelated noise sources.

out this article noise is normalized to per root Hz from equation 1 where  $B = 1$  Hz. Many times the reader may see per Hz, but usually that is an artifact of not having the proper symbol available. Since the square root of 1 is 1, many just ignore this difference even if it is not correct.

### NOISE SOURCES

The need for cross correlation is best illustrated via an example. Assume a very high quality oscillator having a phase noise floor of  $-180$  dBc/Hz with  $+10$  dBm output power. The white noise power of the oscillator is equal to  $10$  dBm ( $-180$  dBc/Hz) or  $-170$  dBm/Hz. A  $50 \Omega$  resistor exhibits about  $0.9 nV/\sqrt{Hz}$  of noise or  $-174$  dBm/Hz in terms of power. A noise power of  $-170$  dBm/Hz is only  $1.44 nV_{rms}/\sqrt{Hz}$ . The best low noise amplifiers are near  $1 nV_{rms}/\sqrt{Hz}$  self noise, limiting both the accuracy and sensitivity of the measurement. This is assuming an ideal phase detector with no loss or noise itself.

To evaluate whether a 'real' mixer could deal with this level of noise, we continue with the example. A  $13$  dBm high quality mixer phase detector may be driven around  $5$  dBm at the RF port. At this level, the mixer is slightly in compression with a conversion loss of likely about  $7$  dB. The output phase conversion may be  $K_d = 0.4$  V/rad or about  $0.282 V_{rms}/rad$ . With a  $50 \Omega$  resistor exhibiting about  $0.9 nV_{rms}/\sqrt{Hz}$ ,

this leaves a dynamic range of  $-170$  dBc/Hz (about  $10$  dB below that of the oscillator). A higher power mixer can increase this dynamic range, but shot noise and other effects can become more dominant (such as the combined impedance of the mixer and the termination at the IF port).

**Table 1** shows the voltage noise of several different resistance values, demonstrating that higher equivalent resistances can significantly degrade the signal to noise ratio. An equivalent port impedance of  $100 \Omega$  can decrease the signal to noise ratio by  $3$  dB.

$$V_n = \sqrt{4kTBR} \quad (1)$$

In an absolute phase noise measurement, the reference oscillator will likely contribute significantly more noise than the mixer, especially at close to the carrier. In an additive measurement such as a low noise, silicon BJT-based RF amplifier, the RF amplifier may have noise that is on par with the silicon diodes used in a good phase detector. In both examples, it is clear that low noise devices require additional techniques to accurately measure them down to very low levels.

### CROSS CORRELATION

A diagram of a two-channel cross-correlation system to measure noise or very small signals is shown in **Figure 1**. Signal  $S_0$  is common while each individual channel has its own independent, and higher, noise. The resultant time domain sampled signal is in the form of  $S_1$  and  $S_2$ . The desired signal to be measured is  $S_0$  and has a small-signal component and broadband noise:

$$S_0 = \alpha \sin(2\pi f t) + v_{n0}(t) \quad (2)$$

The actual measured signal from channels 1 and 2 are defined as

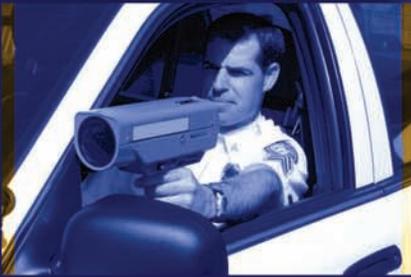
$$S_1 = S_0 + v_{n1}(t) \quad (3)$$

$$S_2 = S_0 + v_{n2}(t) \quad (4)$$

### MATHEMATICAL DEFINITIONS

Cross correlation can be computed via convolution or as a Fourier trans-

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## TECHNICAL FEATURE

form. In the case of spectral analysis, it is easiest to deal with the Fourier transform version. Cross correlation in the time domain is defined by convolution as

$$(S_1 \star S_2) = S_1^*(-t) * S_2 \quad (5)$$

The star is representative of the mathematical function of convolution. The asterisk denotes the complex conjugates of signal  $S_1$ . Rewritten in terms of a Fourier transform, the Fourier transform of the cross correlation is the dot product of the complex conjugate of one Fourier transform to the other Fourier transform.

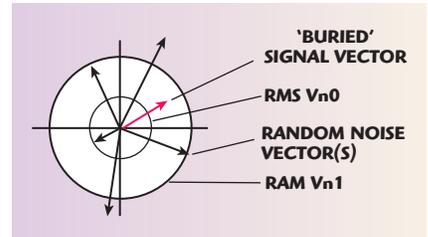
$$F\{S_1 \star S_2\} = F\{S_1\}^* \cdot F\{S_2\} \quad (6)$$

It is shown here that cross correlation makes use of the phase information relative to the two channels. **Figure 2** is a phasor diagram of the magnitude and phase of the Fourier transform for a given frequency component. The red signal vector is the buried signal representative of  $S_0$  while the gray vectors are the random contributions of the whole system noise levels,  $S_1$  or  $S_2$ . The gray vectors are unique and uncorrelated for each channel for each measurement while the red vector is common. Figure 2 illustrates how a buried signal phasor relates to the overall random noise vectors. Two channels will measure the buried signal vector identically with uncorrelated and larger noise in sum with it. Vector averaging the dot product of the two channels (one as a complex conjugate) will reduce the uncorrelated noise proportional to the number of correlations. Vector averaging a number of cross correlations is known as 'cross correlating' with a number of correlations.

The dot product of one signal with the complex conjugate of another for two identical signals results in just the magnitude squared of either signal and is real, having a phase of zero. The dot product of the complex conjugate of one uncorrelated signal to another will be random in both magnitude and phase.

### VECTOR AVERAGING THE CROSS CORRELATION

In a single channel magnitude only system, each frequency bin magnitude is summed and then divided by the number of measurements to



▲ Fig. 2 Phasor diagram of the vector Fourier component of any given frequency.

**TABLE II**

**CROSS CORRELATIONS CAN IMPROVE THE SENSITIVITY IN dB OF IDENTIFYING A SMALL-SIGNAL BY THE RELATION  $5\log_{10}(N)$  WHERE N IS THE NUMBER OF CORRELATIONS**

Correlations (N)	Noise Floor Improvement (dB)
1	0
10	5
100	10
1000	15
10000	20

achieve averaging. In a cross-correlation system, both the magnitude and phase, or real and imaginary parts of the dot product from the cross correlation are vector summed. The dot product of the random noise vectors will eventually achieve a vector sum of zero assuming they are truly random and uncorrelated. The dot product of the small common signal will be in phase and real and eventually be the only signal left.

Vector averaging yields a maximum improvement relative to the number of averages (N) in dB as  $5\log_{10}(N)$ . In other words, it takes an order of magnitude increase in measurements for every 5 dB improvement. **Table 2** lists the improvement for a number of correlations.

### EXAMPLE

To illustrate how cross correlation works, a mathematical equivalent of Figure 1 was defined in numerical computing software using the signals  $S_0$ ,  $S_1$  and  $S_2$ .  $S_0$  represents a small-signal above a broadband low noise level equivalent to just over a 50  $\Omega$  noise floor.  $S_1$  and  $S_2$  contain noise components  $v_{n1}$  and  $v_{n2}$ , respectively, with noise levels that are above that of the small-signal of  $S_0$ . The example demonstrates that a single channel system could never measure  $S_0$ , given

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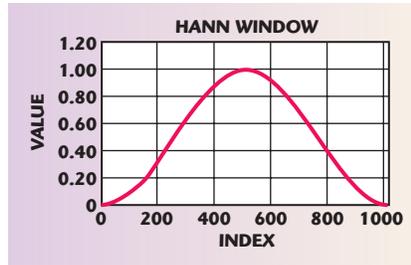
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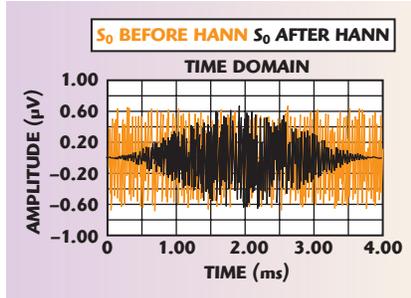


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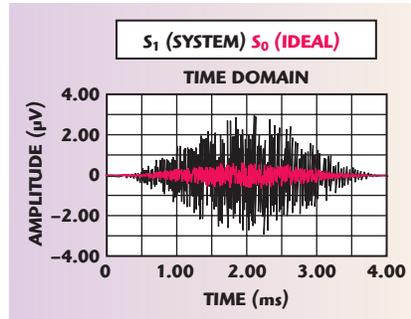
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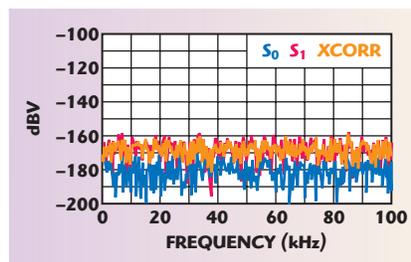
▲ Fig. 3 Hann window filter to be applied to a 1024 point time domain sample.



▲ Fig. 4 Time domain signal of  $S_0$  before and after application of a Hann filter.



▲ Fig. 5 Time domain of 'system'  $S_1$  with superimposed time domain of desired signal.

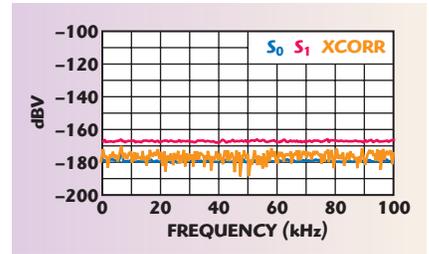


▲ Fig. 6 The three traces represent signal  $S_0$ ,  $S_1$ , and the cross-correlation of  $S_0$  and  $S_1$  with 1 correlation.

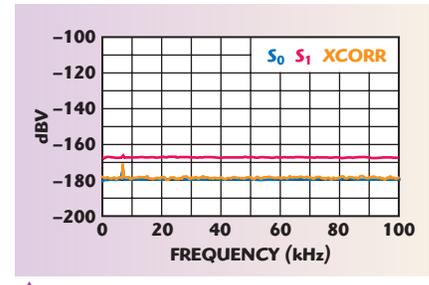
the noise levels of the receiver with resultant signal  $S_1$ .

### WINDOWING

Window filtering is an important step in noise analysis. Discontinuities that occur during the start and stop of the finite sampling of a signal can cause spectral-leakage from one frequency to another, once the Fourier transform is applied. In low dynamic range systems,



▲ Fig. 7 The three traces represent signal  $S_0$ ,  $S_1$ , and the cross-correlation of  $S_0$  and  $S_1$  with 100 correlations.



▲ Fig. 8 The three traces represent signal  $S_0$ ,  $S_1$ , and the cross-correlation of  $S_0$  and  $S_1$  with 10,000 correlations.

such as a 6- or 8-bit oscilloscope, this may not have much of an effect. However, with higher dynamic range systems (>60 dB), this can cause significant problems. To overcome this, the input time domain signal is windowed with an additional function to reduce this effect. For this example, we use the Hann window shown in **Figure 3**, one of the more common windows used with noise analysis. The Hann window is defined as

$$\omega(n) = \frac{1}{2} \left( 1 - \cos \left( \frac{2\pi n}{N-1} \right) \right) \quad (7)$$

where  $N$  is the number of time domain samples. In this example we use 1024 points. Windowing of the ideal signal  $S_0$  is shown in **Figure 4**. In a real (noisy) measurement system, with noise components  $v_{n1}$  and  $v_{n2}$  being greater than signal  $S_0$ , the actual measurement is going to look more like **Figure 5**. For comparison, **Figure 5** shows the ideal  $S_0$  superimposed with the measured system signal  $S_1$ .

At this point, the cross-correlation algorithm is applied to signals  $S_1$  and  $S_2$  with vector averaging (cross correlations) at 1, 100 and 10,000 for a maximum improvement of 0, 10 and 20 dB, respectively. **Figures 6, 7** and **8** demonstrate the effectiveness of cross correlation for 1, 100 and 10k correlations, respectively. Each plot contains three curves. The blue curve is the

**TABLE III**

**RESULTS OF THE EXAMPLE SHOWING AN IMPROVEMENT OF MEASUREMENT SENSITIVITY WITH A HIGHER NUMBER OF CORRELATIONS**

<i>XCORR</i>	<i>Input Noise (dBc/√Hz)</i>	<i>LNA (dBV/√Hz) Noise</i>	<i>Measured Noise (dBV/√Hz)</i>
1	-179	-167	-167
100	-179	-167	-176
10k	-179	-167	-178.5

ideal signal  $S_0$ , having the number of correlations equal to the number of averages. The red curve is the signal  $S_1$  of the individual channel, having the number of correlations equal to the number of averages. The yellow curve is the resultant cross correlation after vector averaging the number of 'correlations'. The results are tabulated in **Table 3**, respective of the number of correlations.

**CONCLUSION**

Cross-correlation analysis trades measurement setup complexity and

time for increased sensitivity. In situations such as measuring phase noise where hardware can limit the measurement sensitivity compared to the device under test, cross correlation may be applied to improve the sensitivity to acceptable levels. In cases where the measurement floor is still below the device under test, but very close, cross correlation will improve the overall accuracy of the measurement. For cross correlation to be most effective, each channel must be isolated as much as possible to reduce or eliminate common mode noise. Additionally, the

lower the noise measurement capability of each individual channel, the fewer correlations will be required and the faster a measurement can occur. As shown in the example plots of Figures 6-8, cross correlation has the ability to pick up noise and small signals that are otherwise invisible to traditional measurement systems. ■

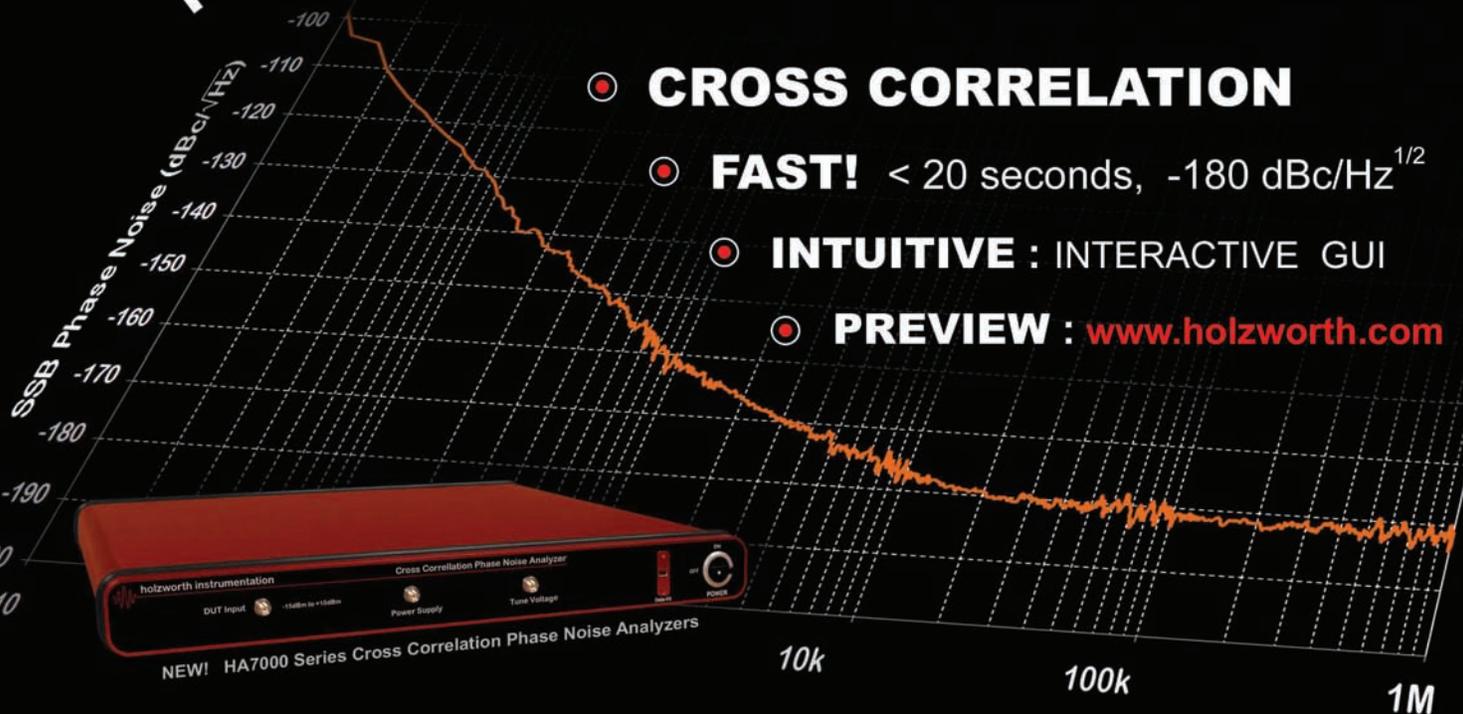
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